Verifying dependent type-checkers

WITS 2026

Meven Lennon-Bertrand

I've written one type-checker in my life, and for a rather simple theory...

A disclaimer

I’ve written one type-checker in my life, and for a rather simple theory…

so I’m going to do **propaganda**!

Because that type-checker was **fully verified!**

Verifying the verifier

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A lot of the verification ecosystem relies on proof assistant kernels/dependent type-checkers...

Why don't we still have verified kernels?

The programs are not that complicated...

But the reasons why they work are complicated!

Logical relations

No computational content

Meta-theory

Models

Algebraic

Presentation-free

Execution/Extraction

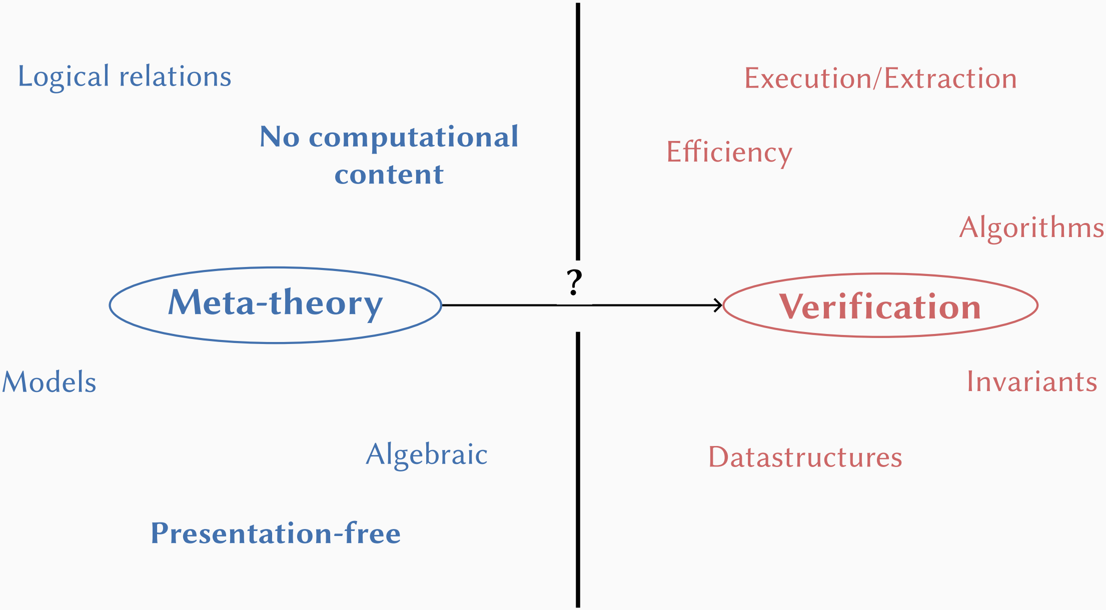
Efficiency

Algorithms

Verification

Invariants

Datastructures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All hail verification

Much focus on the **meta-theory** side

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But a lot of interesting questions on the **verification** side:

- What is the abstract specification of your type system?
- How do the datastructures you use relate to their abstract variants?
- What invariants do your code & datastructures rely on?
- What meta-theoretic properties are needed to verify these invariants?

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But a lot of interesting questions on the **verification** side:

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Particularly worth asking for **realistic implementations**

We should **verify implementations** even if **we don't do the meta-theory**

This talk

We should **verify implementations** even if **we don't do the meta-theory**

A tour of case studies in that space:

- pattern-matching in MetaRocq
  - *Correct and Complete Type Checking and Certified Erasure for Coq, in Coq* (JACM 2025)
  - *The Curious Case of Case: Correct & Efficient Representation of Case Analysis in Coq and MetaCoq* (WITS 2022)
- verifying untyped conversion
  - *Martin-Löf à la Coq* (CPP 2024)
  - *What Does It Take to Certify a Conversion Checker?* (FSCD 2025)

MetaRocq: the curious case of case

The Predicative Calculus of Universe-Polymorphic Inductive Constructions (PCUIC)

A dependent type theory with

- Crazy (co-)inductive types
- Pattern-matching and fixed-points
- Fancy universes + cumulativity

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Rocq, in Rocq

- **Formalised meta-theory of PCUIC**
- Normalisation axiom to implement a **verified type-checker**
- **Verified extraction**
- Meta-programming

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We found a bug in Rocq!

Coq 6.1	Pattern-matching (Cornes), representation chosen for backwards compatibility
Coq 8.4	Universe polymorphism (Sozeau & Tabareau)
Coq 8.7	Cumulative inductive types, theory **for eliminators** (Sozeau & Timany)
Meanwhile	People like less and less the clunky pattern-matching representation

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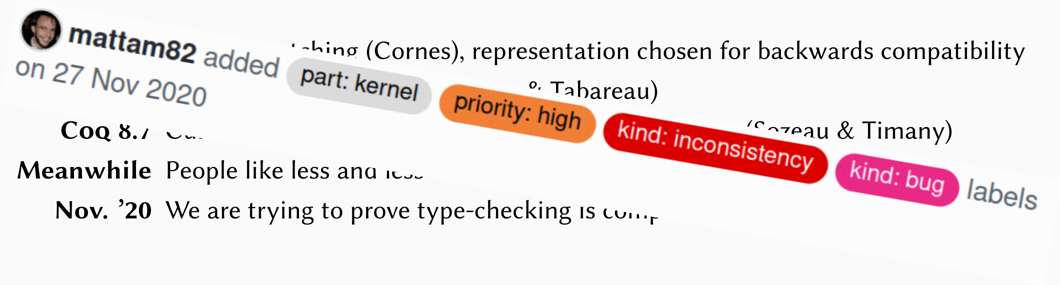

Post-mortem of a bug

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Coq 8.13	**Kernel bug**! ⟶ quick and dirty fix
Coq 8.14	Complete redesign, in parallel in Rocq and MetaRocq

Issue #13495

```
match s as x in Ind _ inds return P with
…
end
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6.1 – 8.13:

1. infer the type Ind $\vec{p}\,\vec{i}$ of s
2. "check" P against $\Pi(\overrightarrow{inds} : \mathrm{Indices}_{\mathrm{Ind}}[\vec{p}], x : \mathrm{Ind}\ \vec{p}\ \overrightarrow{inds}).\, \square_?$
3. check the branches
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Subtle **implementation mis-design**, on a **combination of advanced features**
Never described on paper or formalised

What do we want to verify?
What does it take?

Declarative specification

Arbitrarily mixing:

- Refl./Sym./Trans.
- Congruences
- Computation (β)
- Extensionality (η)

Typed!

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Type-directed algo.

Alternate

1. β-reduction to whnf
2. **Type**-directed η
3. **Head** congruences

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1. β-reduction to whnf
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+ closer to specification
+ supports fancier rules
- slower

“Untyped” algo.

Alternate

1. β-reduction to whnf
2. Term-directed η
3. Head congruences

+ faster
+ simpler
- further from spec.

What to verify?

$$P : D \to \mathbb{P} \quad \overset{?}{\Leftrightarrow} \quad p : D \to \mathbb{B}$$

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0. decidability: $(p\ d = \text{true}) \vee (p\ d = \text{false})$

1. soundness: $p\ d = \text{true} \Rightarrow P\ d$

2. completeness: $P\ d \Rightarrow p\ d = \text{true}$

3. profit!

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   Look at the trace of the type-checker
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2. **completeness**: $P\ d \Rightarrow p\ d = \text{true}$
   reflexivity ⇒ termination
3. profit?

What to verify?

$$P : D \to \mathbb{P} \qquad \overset{?}{\Leftrightarrow} \qquad p : D \rightharpoonup \mathbb{B}$$

1. **positive soundness**: $p\ d = \text{true} \Rightarrow P\ d$
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2. **negative soundness**: $p\ d = \text{false} \Rightarrow \neg\,(P\ d)$
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A much better plan

What meta-theory do we need?

	Positive soundness	Negative soundness (typed conversion)	Negative soundness (untyped conversion)	Termination
Injectivity of type constructors	×	×	×	×
Term-level injectivities		×	×	
Normalisation				×

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Injectivities are the important properties

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Injectivities are the important properties

To verify "untyped" conversion, you still need typing invariants

Injectivity of type constructors

If $\Gamma \vdash T \equiv T'$ and T, T' are weak-head normal form, then:

- $T = \mathbb{N} = T'$
- or $T = \Pi\, x\colon A.B$, $T' = \Pi\, x\colon A'.B'$, with $\Gamma \vdash A' \equiv A$ and $\Gamma, x\colon A' \vdash B \equiv B'$
- or ...
- or T, T' are both neutrals, and $\Gamma \vdash T \equiv T' : \square$

Any non-diagonal case is impossible (*no-confusion*).

Injectivity of type constructors

Injectivity and no-confusion at $\mathbb{N}$

If $\Gamma \vdash n \equiv n' : \mathbb{N}$ and n, n' are weak-head normal forms, then:

- $n = 0 = n'$
- or $n = \mathrm{S}(t)$, $n' = \mathrm{S}(t')$, with $\Gamma \vdash t \equiv t' : \mathbb{N}$
- or n, n' are both neutrals.

Injectivity of type constructors

Injectivity and no-confusion at $\mathbb{N}$

…

Normalisation

Inductive predicate: "iterated weak-head normalisation and η-expansion terminates".

Understanding "untyped" conversion-checking

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Injectivities are the important properties

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There is a catch on neutrals!

Neutrals

Traditional PL: evaluation of closed, first-order values
Dependent types: conversion under binders, must consider **open terms**

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A neutral is:

- a variable
- or an elimination, stuck on a neutral

$x : \mathbb{N} \to \mathbb{N} \times \mathbb{N}, \ldots \vdash \mathrm{rec}_{\mathbb{N}}(\pi_1(x\ 7), P, b_0, b_S) : P$

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How does one compare neutrals? Where does one use neutral comparison?

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Type-directed conversion

$$\frac{}{x : \mathbb{N} \to \mathbb{N} \vdash x \equiv x : \mathbb{N} \to \mathbb{N}}$$

Type-directed conversion

$$\frac{x\colon \mathbb{N} \to \mathbb{N}, y\colon \mathbb{N} \vdash x\ y \equiv x\ y : \mathbb{N}}{x\colon \mathbb{N} \to \mathbb{N} \vdash x \equiv x : \mathbb{N} \to \mathbb{N}}$$

Type-directed conversion

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Term-directed conversion

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Why does this work?

Injectivity of neutral eliminators?

If $\Gamma \vdash n \equiv n' : T$ and n and n' are neutrals, then

- $n = x = n'$
- or $n = m\ u, n' = m'\ u'$ with $\Gamma \vdash m \equiv m' : \Pi\, x{:}\, A.B$ and $\Gamma \vdash u \equiv u' : A$
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Does not always hold!

$$x, y{:}\, (\mathbb{N} \to \mathbb{1}) \times \mathbb{1} \vdash x \equiv y : (\mathbb{N} \to \mathbb{1}) \times \mathbb{1}$$

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Neutral comparison is complete only at certain types

Agda
Type-directed
Short path for neutral functions

Lean
Term-directed
Detect unit-like types

Rocq
Term-directed
Forbid unit-like types

Wrapping up

Verification is very useful!

- Bug finding
- Looking hard at the dark corners
- Uncover & understand assumptions hidden in implementation subtleties

Looking back

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- Reduce folklore
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But: the cost is currently quite high…

Lots of cool things happening in the space:

AgdaCore	Agda-style pattern-matching + termination-checker
Lean4Lean	essentially the real kernel's code, with a dedicated program logic
MetaRocq	algebraic universes, nested inductive types...
λ□	whole ecosystem of verified compilations
And more	LogRel-Rocq, LogRel-MLTT/`graded-type-theory`, McTT, Liu & Weirich...

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- Reusable libraries and insight
- Automation (AutoSubst/Sulfur)
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How do we bridge the gap with meta-theory-oriented formalisations?

We can (should!) separate **meta-theory** and **verification**

Find the right interfaces, from **synthetic methods** to **MetaRocq and beyond**

A lot is happening, **stay tuned**, or better: **come join us!**

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Thank you!

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