An Attempt at Necromancy

Experience Report on Modernising a Domain Theory Library

Rocqshop – July 25th 2026

Meven Lennon-Bertrand

INRIA – IRIF, Université Paris Cité

of "inclusive predicates", fundamental in domain theory [12, 15]. Another technical advantage of our approach is that we don't need to use contexts as Kripke worlds as in previous arguments [2, 6]. We also establish that two convertible terms in type theory (maybe *partial* [10, 11, 13, 14]) have the same Böhm tree.

In this paper, we work in a constructive metatheory, and when we write that a propoosition P is decidable, we mean that $P \vee \neg P$ is provable.

2 Domain and Finite Elements

We shall use the following Scott domain, least solution of a recursive domain equation (see [18, 19] for a lively description of Scott domains and solutions to domain equations):

$$D \;=\; [D \to D] + \Pi\ D\,[D \to D] + \mathsf{N} + \mathsf{0} + \mathsf{S}\ D + \mathsf{U}_i$$

In this equation, $+$ denotes the coalesced sum [18] and $i = 0, 1, 2, \ldots$

We write $a, b, u, v, \ldots$ for the elements of this domain. We define $u(v)$ for u and v in D as follows: it is the application of u to v if u belongs to $D \to D$ and it is $\perp$ otherwise.

An Adequacy Theorem for Dependent Type Theory (Coquand et al. 2018)

Formalized, Effective Domain Theory in Coq

Robert Dockins

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Abstract. I present highlights from a formalized development of domain theory in the theorem prover Coq. This is the first development of domain theory that is *effective*, *formalized* and that supports all the usual constructions on domains. In particular, I develop constructive models of both the unpointed profinite and the pointed profinite domains. Standard constructions (e.g., products, sums, the function space, and powerdomains) are all developed. In addition, I build the machinery necessary to compute solutions to recursive domain equations.

1 Introduction and Related Work

The term "domain theory" refers to a class of mathematical techniques that are used to develop computational models suitable for reasoning about the semantics of general purpose programming languages. Proofs done in domain theory often have an elegant, compositional nature, and the much of the modern thinking about programming languages (especially functional languages) can be traced to domain-theoretic roots. Domain theory has a long lineage, starting from the classic work of Dana Scott on continuous lattices [22]. Domain theory was intensively studied in the 1970's and 80's, producing far more research than I have space here to review; see Abramsky and Jung's account for a good overview and further sources [2].

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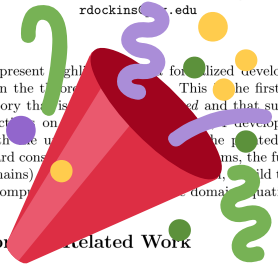

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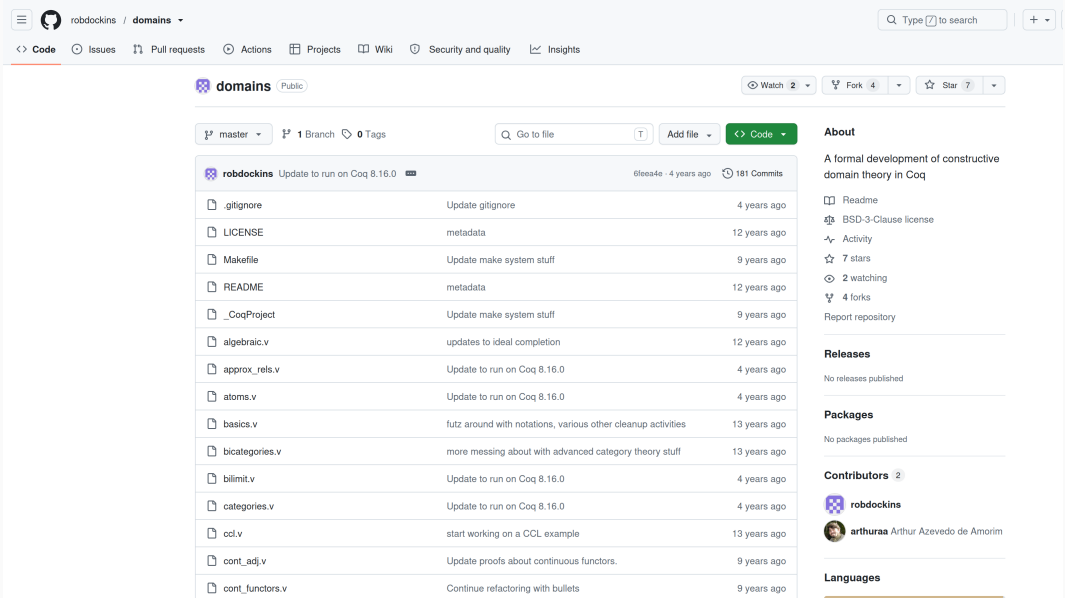
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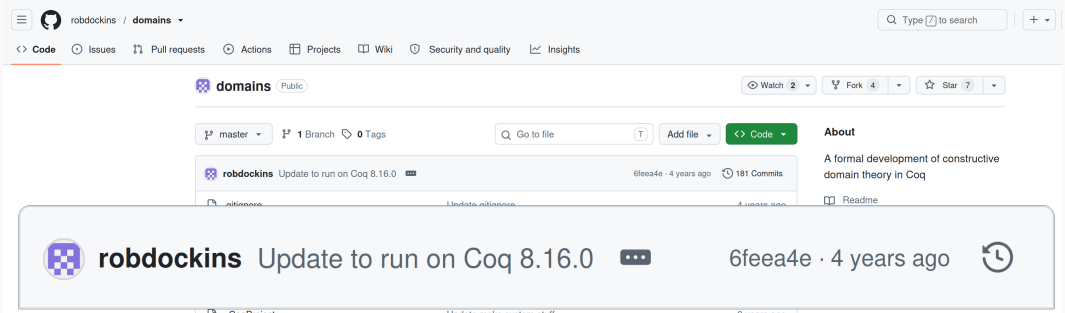
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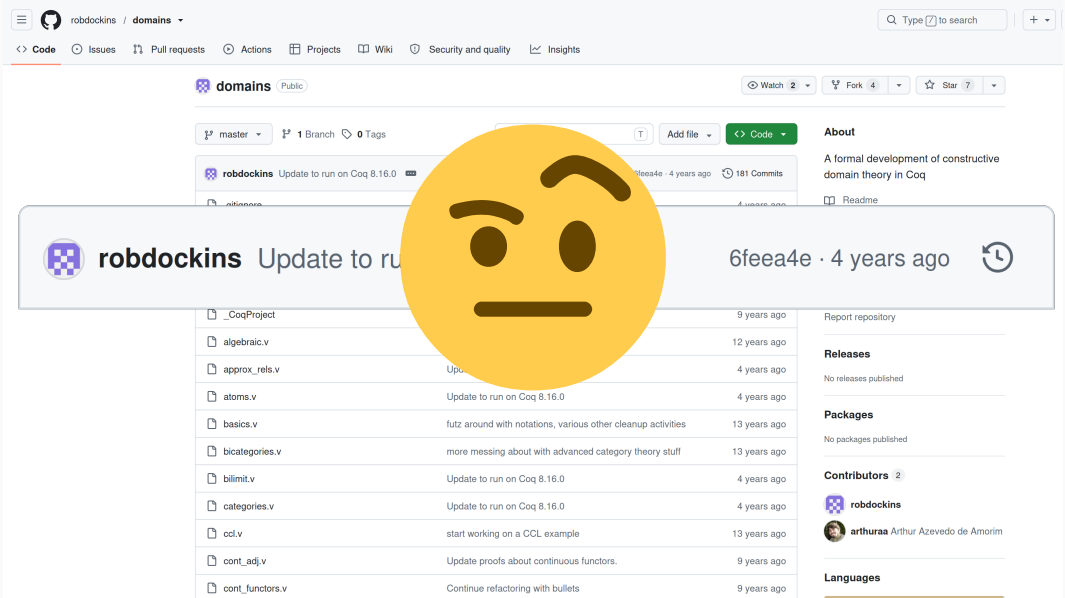
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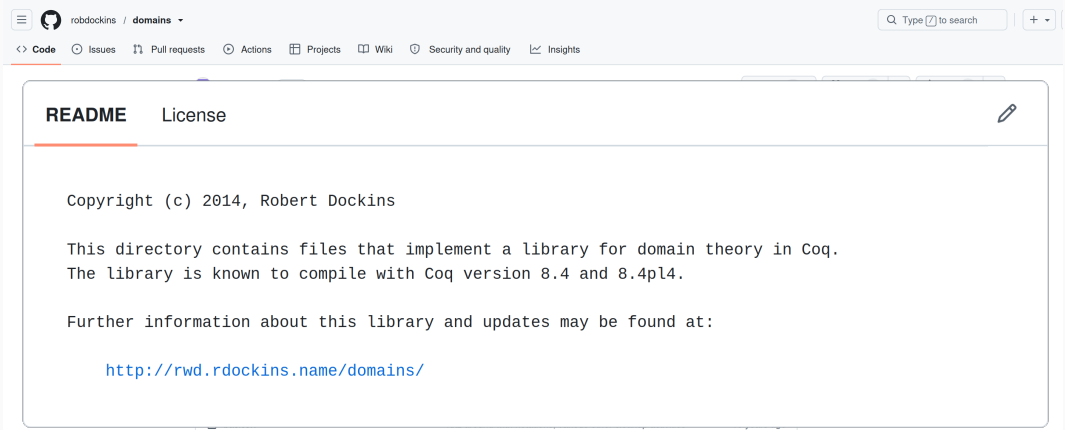

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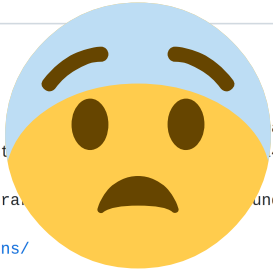
This directory contains files that ain theory in Coq.
The library is known to compile wit 4.

Further information about this libra und at:

http://rwd.rdockins.name/domains/



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Languages

The plan

1. Some reflexions on porting
2. Some technical reflexions

The plan

1. **Some reflexions on porting**

2. **Some technical reflexions**

More food for thought than very conclusive
⟶ Let's chat!

Some reflexions on porting

Coq 8.16 → Rocq 9.1

It was a breeze!

Coq 8.16 → Rocq 9.1

It was a breeze! But...

```
destruct (finsubset_dec' A (OrdDec A (eff_ord_dec A Heff)) P') with M; auto.
- intro x. unfold P'.
  destruct (inh_dec A hf x).
  + destruct (finset_find_dec' A (fun p:A ⇒ p ∈ M)) with x; simpl.
    * intros. rewrite <- H0; auto.
    * intros. apply finset_in_dec.
      constructor. apply eff_ord_dec. auto.
    * left. intros Hx ?.
      destruct s. destruct a.
      apply H0 in H1. elim H2; auto.
    * destruct (normal_has_mubs Q HQ x) as [MUBS [?[??]]]; auto.
      ** red; intros. apply HM. apply m. auto.
      ** destruct (finset_find_dec' A (fun p ⇒ p ∈ M)) with MUBS; simpl.
          [ ... ]
          *** left. intros _. intros.
              apply m0.
              destruct (H2 x0) as [x0' [?[??]]].
              **** destruct H4; auto.
```

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What does it mean to “port a library to Rocq 9.1”?

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- foundations?
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Libraries will decay

Is long-term backwards compatibility that important?

Theory building

Programming as Theory Building (Naur 1985)

Theory building

A person who has or possesses a theory in this sense knows how to do certain things and in addition can support the actual doing with explanations, justifications, and answers to queries, about the activity of concern.
[...]
The programmer having the theory of the program can explain why each part of the program is what it is, in other words is able to support the actual program text with a justification of some sort.

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The death of a program happens when the programmer team possessing its theory is dissolved. A dead program may continue to be used for execution in a computer and to produce useful results. [...] Revival of a program is the rebuilding of its theory by a new programmer.

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Please don't tell me about AI

An attempt at necromancy

The new programmer is likely to feel torn between loyalty to the existing program text, with whatever obscurities and weaknesses it may contain, and the new theory that [they have] to build up, and which [...] will differ from the original theory behind the program text.

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Concretely: is this design decision

- good (although maybe I can't appreciate why)?
- bad (and I can/should change it)?
- historically good but not valid any more?

What to keep?

In preference to program revival, the Theory Building View suggests, the existing program text should be discarded and the new-formed programmer team should be given the opportunity to solve the given problem afresh.

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My approach: original mechanisation = very detailed mathematical text to mechanise

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It's nice to run the original proof script

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Think about this when mechanising & writing/reviewing papers!

Some technical reflexions

Setoids, setoids everywhere

```
Record theory (set : preord -> Set) := {
member : forall (A:preord), A -> set A -> Prop.
member_eq : forall (A:preord) (a b:A) (X:set A),
    a ≈ b -> member A a X -> member A b X;
… }.

Definition fset (A:preord) := list A.
Definition fmember (A:preord) (a:A) (P:fset A) := exists a', In a' P ∧ a ≈ a'.
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Small tactic computes identity types:

```
f, g : nat -> {n : nat & P n}
═══════════════════════════════
f = g
```

->

```
f, g : nat -> {n : nat & P n}
x : nat
═══════════════════════════════
proj1_sig (f x) = proj1_sig (g x)
```

Choosing is always a problem

Vanilla setoid model validates little choice...

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Issue 1: Less flexibility than with (analytic) setoids

Countable union of countable sets = fine with setoids, countable choice synthetically

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Is there a better model? Is it worth losing the intuitive setoid reading?

Hierarchy builder is very very nice!

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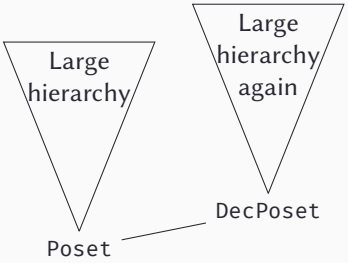

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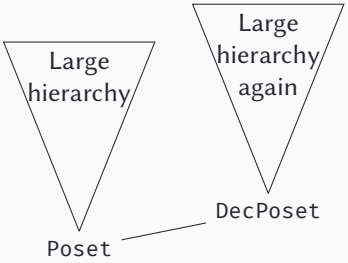

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"Complicated" inference patterns:

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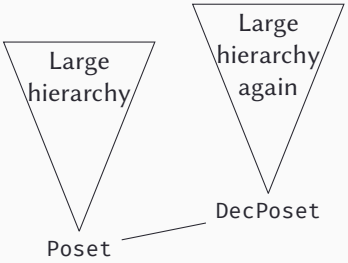

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Type-classes ? Some sort of combination? A new mechanism?

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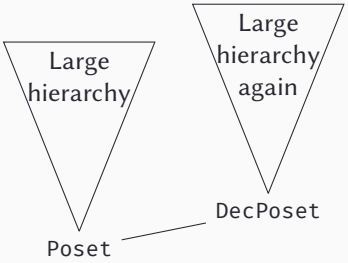

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Also, in practice: lots of layers that can go wrong (and do...)

Wrapping up

Go read *Programming as Theory Building*

The future is coming, but there's still some way to go

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Thank you!