Injectivity of type constructors

Implementation of type theory seminar – March 10th 2026

What am I trying to do?

Verifying the verifier

We keep telling the world they should verify their critical code...

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The programs are (relatively) simple...

But the reasons why they work are complicated!

Where I'm coming from

- MetaRocq: Rocq in Rocq
- *Martin-Löf à la Coq*: a place to experiment

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Today: some things I've learned to care about there
Main reference: *What Does It Take to Certify a Conversion Checker?* (FSCD 2025)

Logical relations

Irrelevant
computational content

Meta-theory

Models

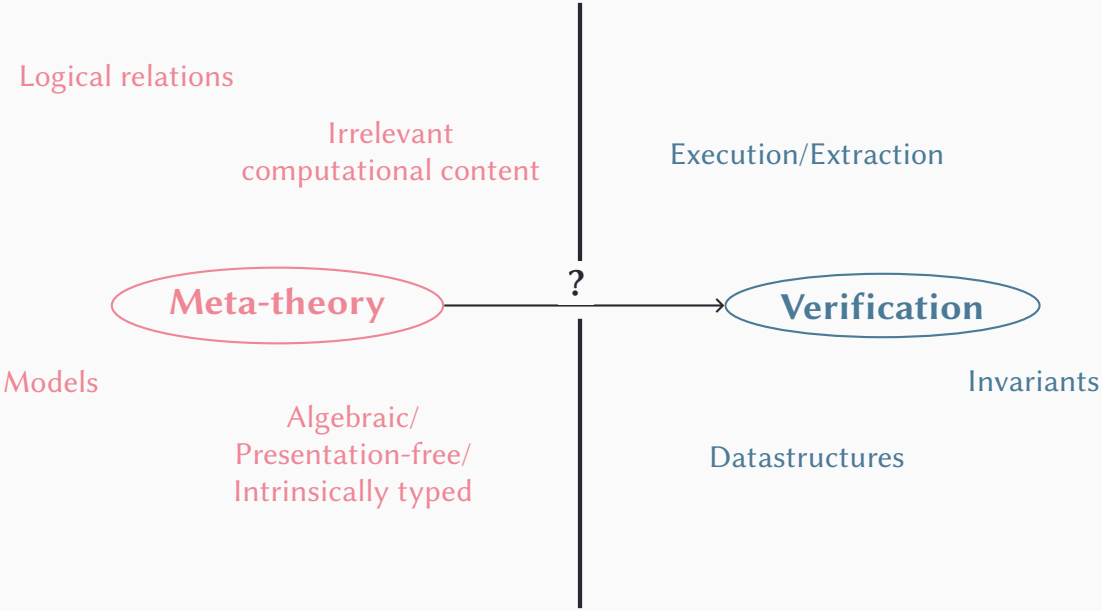
Algebraic/
Presentation-free/
Intrinsically typed

Execution/Extraction

Verification

Invariants

Datastructures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The algorithms (and their specification)

Specification

Rules for each term/type former
$(\Pi, \Sigma, \mathrm{Id}, \mathcal{U}, \mathbb{N}, \bot \ldots)$ +

$$\frac{\Gamma \vdash t : T \qquad \Gamma \vdash T \cong T'}{\Gamma \vdash t : T'}$$

Specification

Rules for each term/type former $(\Pi, \Sigma, \mathrm{Id}, \mathcal{U}, \mathbb{N}, \bot \ldots)$ +

$$\frac{\Gamma \vdash t : T \qquad \Gamma \vdash T \cong T'}{\Gamma \vdash t : T'}$$

Algorithm

Bidirectional: rules for each term former integrate (some) conversion.

Information Must Flow

Inference and checking

$\Gamma \vdash t : A$ separates into

inference: $\Gamma \vdash t \triangleright A$ checking: $\Gamma \vdash t \triangleleft A$

Similar “meaning”, different operational modes: input/subject/output.

Structure!

$$\frac{\Gamma \vdash t : \Pi\, x{:}\, A.B \qquad \Gamma \vdash u : A}{\Gamma \vdash t\ u : B[u]}$$

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$$\frac{\Gamma \vdash t \triangleright_{\mathrm{h}} \Pi\, x{:}\, A.B \qquad \Gamma \vdash u \triangleleft A}{\Gamma \vdash t\ u \triangleright B[u]}$$

- Clear **information flow**

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$$\frac{\Gamma \vdash t : T \qquad \Gamma \vdash T \cong T'}{\Gamma \vdash t : T'}$$

- Clear information flow
- ~~Unconstrained conversion~~ $\longrightarrow$ Computation ($\rightsquigarrow^\star$ or $\cong$) **guided by the mode**

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$$\frac{\Gamma \vdash t : T \qquad \Gamma \vdash T \cong T'}{\Gamma \vdash t : T'} \qquad \frac{\Gamma \vdash t \triangleright T \qquad \Gamma \vdash T \cong T' \triangleleft}{\Gamma \vdash t \triangleleft T'} \qquad \frac{\Gamma \vdash t \triangleright T \qquad T \rightsquigarrow^{\star} T'}{\Gamma \vdash t \triangleright_{\mathrm{h}} T'}$$

- Clear information flow
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Similar "meaning", different operational modes: input/subject/output.

McBride: ***A rule is a server for its conclusion and a client for its premises.***

- Modes guide invariant preservation
- In a conclusion, you assume inputs are well-formed, and ensure outputs are
- In a premise, you ensure inputs are well-formed, and assume outputs are

Declarative specification

Arbitrarily mixing:

- Refl./Sym./Trans.
- Congruences
- Computation (β)
- Extensionality (η)

Typed!

Conversion

Declarative specification

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Typed!

Type-directed algo.

Alternate

1. β-reduction to whnf
2. **Type**-directed η
3. **Head** congruences

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- Computation (β)
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Typed!

Type-directed algo.

Alternate

1. β-reduction to whnf
2. Type-directed η
3. Head congruences

\+ closer to specification
\+ supports fancier rules
\- slower

“Untyped” algo.

Alternate

1. β-reduction to whnf
2. Term-directed η
3. Head congruences

\+ faster
\+ simpler (?)
\- further from spec.

Type-directed conversion

$$x : \mathbb{N} \to \mathbb{N} \vdash x \cong x : \mathbb{N} \to \mathbb{N}$$

Type-directed conversion

$$\dfrac{\dfrac{}{x{:}\,\mathbb{N} \to \mathbb{N},\, y{:}\,\mathbb{N} \vdash x\ y \cong x\ y : \mathbb{N}}}{x{:}\,\mathbb{N} \to \mathbb{N} \vdash x \cong x : \mathbb{N} \to \mathbb{N}}$$

Type-directed conversion

$$\dfrac{\dfrac{}{x\colon \mathbb{N} \to \mathbb{N},\, y\colon \mathbb{N} \vdash x\ y \cong x\ y : \mathbb{N}}}{x\colon \mathbb{N} \to \mathbb{N} \vdash x \cong x : \mathbb{N} \to \mathbb{N}}$$

Type-directed conversion

$$\dfrac{\dfrac{}{x\colon \mathbb{N} \to \mathbb{N}, y\colon \mathbb{N} \vdash x\ y \approx x\ y : \mathbb{N}}}{x\colon \mathbb{N} \to \mathbb{N}, y\colon \mathbb{N} \vdash x\ y \cong x\ y : \mathbb{N}}}{x\colon \mathbb{N} \to \mathbb{N} \vdash x \cong x : \mathbb{N} \to \mathbb{N}}$$

Type-directed conversion

$$\dfrac{\dfrac{(x\colon \mathbb{N} \to \mathbb{N}) \in x\colon \mathbb{N} \to \mathbb{N}, y\colon \mathbb{N}}{x\colon \mathbb{N} \to \mathbb{N}, y\colon \mathbb{N} \vdash x \approx x : \mathbb{N} \to \mathbb{N}} \qquad \dfrac{\ldots}{x\colon \mathbb{N} \to \mathbb{N}, y\colon \mathbb{N} \vdash y \cong y : \mathbb{N}}}{\dfrac{x\colon \mathbb{N} \to \mathbb{N}, y\colon \mathbb{N} \vdash x\ y \approx x\ y : \mathbb{N}}{\dfrac{x\colon \mathbb{N} \to \mathbb{N}, y\colon \mathbb{N} \vdash x\ y \cong x\ y : \mathbb{N}}{x\colon \mathbb{N} \to \mathbb{N} \vdash x \cong x : \mathbb{N} \to \mathbb{N}}}}$$

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Term-directed conversion

$$\dfrac{\dfrac{}{x \approx x}}{x \cong x}$$

When is this valid?

The plan

What's in a decision procedure?

$$P : D \to \mathbb{P} \qquad \overset{?}{\Leftrightarrow} \qquad p : D \to \mathbb{B}$$

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2. completeness: $P\ d \Rightarrow p\ d = \text{true}$

3. profit!

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How do you know that the type-checker terminates?

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   How do you know that the type-checker terminates?
1. **soundness**: $p\ d = \text{true} \Rightarrow P\ d$
   Look at the trace of the type-checker
2. completeness: $P\ d \Rightarrow p\ d = \text{true}$

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2. **completeness**: $P\ d \Rightarrow p\ d = \text{true}$
   reflexivity $\simeq$ normalisation
3. profit?

Better abstract nonsense

$$P : D \to \mathbb{P} \qquad \overset{?}{\Leftrightarrow} \qquad p : D \rightharpoonup \mathbb{B}$$

1. **positive soundness**: $p\ d = \text{true} \Rightarrow P\ d$
   Look at the trace of the type-checker

Better abstract nonsense

$$P : D \to \mathbb{P} \qquad \overset{?}{\Leftrightarrow} \qquad p : D \rightharpoonup \mathbb{B}$$

1. **positive soundness**: $p\ d = \text{true} \Rightarrow P\ d$
   Look at the trace of the type-checker
2. **negative soundness**: $p\ d = \text{false} \Rightarrow \neg(P\ d)$
   Look (harder) at the trace of the type-checker

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Now we have a plan

What meta-theory do we need?

Neutrals and normals

The least amount of information on a term that is stable by conversion

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(Weak-head) normal forms

Canonical forms

$$\boxed{\mathrm{can}\ f} \quad \stackrel{\text{def}}{=} \quad \mathcal{U} \mid \Pi\, x{:}t.t \mid \mathbb{N} \mid \Sigma\, x{:}t.t \mid \lambda\, x{:}t.t \mid 0 \mid \mathrm{S}(t) \mid (t,t) \mid \ldots$$

Neutrals and normals

The least amount of information on a term that is stable by conversion

(Weak-head) normal forms

Canonical forms, or neutrals

$$\boxed{\mathrm{can}\ f} \quad \stackrel{\text{def}}{=} \quad \mathcal{U} \mid \Pi\, x{:}t.t \mid \mathbb{N} \mid \Sigma\, x{:}t.t \mid \lambda\, x{:}t.t \mid 0 \mid \mathrm{S}(t) \mid (t,t) \mid \ldots$$

$$\boxed{\mathrm{nf}\ f} \quad \stackrel{\text{def}}{=} \quad \mathrm{can}\ f \vee \mathrm{ne}\ f$$

(Weak-head) neutrals

Elimination, blocked on a variable

$$\boxed{\mathrm{ne}\ n} \quad := \quad x \mid n\ t \mid \pi_1\ n \mid \pi_2\ n \mid \mathrm{rec}_{\mathbb{N}}(n, x.t, t, x.y.t) \mid \ldots$$

$x{:}\mathbb{N} \to \mathbb{N} \times \mathbb{N}, \ldots \vdash \mathrm{rec}_{\mathbb{N}}(\pi_1(x\ 7), P, b_0, b_S) : P$

Injectivity and no-confusion of type constructors

If $\Gamma \vdash T \cong T'$ and T, T' are weak-head normal form, then:

- $T = \mathbb{N} = T'$
- or $T = \Pi\, x{:}\, A.B$, $T' = \Pi\, x{:}\, A'.B'$, with $\Gamma \vdash A' \cong A$ and $\Gamma, x{:}\, A' \vdash B \cong B'$
- or ...
- or T, T' are both neutral, and $\Gamma \vdash T \cong T' : \mathcal{U}$

Any non-diagonal case is impossible (*no-confusion*).

Note: easy to break! Even in a consistent system!

Injectivity and no-confusion of type constructors

Injectivity and no-confusion at $\mathbb{N}$

If $\Gamma \vdash n \cong n' : \mathbb{N}$ and n, n' are weak-head normal forms, then:

- $n = 0 = n'$
- or $n = \mathrm{S}(t), n' = \mathrm{S}(t')$, with $\Gamma \vdash t \cong t' : \mathbb{N}$
- or n, n' are both neutral.

The good properties

Injectivity and no-confusion of type constructors

Injectivity and no-confusion at $\mathbb{N}$

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...

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Injectivity of neutral eliminators*

If $\Gamma \vdash n \cong n' : T$ and n and n' are neutrals, then

- $n = x = n'$
- or $n = m\ u, n' = m'\ u'$ with $m \cong m'$ and $u \cong u'$
- or $n = \mathrm{rec}_{\mathbb{N}}(m, x.P, t_0, x.y.t_{\mathsf{S}}), n' = \mathrm{rec}_{\mathbb{N}}(m', x.P', t_0', x.y.t_{\mathsf{S}}')$, and ...

* More on this in a moment

The good properties

Injectivity and no-confusion of type constructors

Injectivity and no-confusion at $\mathbb{N}$

Injectivity and no-confusion at $\mathcal{U}$

Injectivity of neutral eliminators*

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Deep normalisation

Every well-typed term is deeply normalising at its type.
Every well-formed type is deeply normalising.

Positive soundness

Should be easy, right? All algorithmic rules are also valid declaratively and tada!

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Invariants: *A rule is a server for its conclusion and a client for its premises*

+ no need to re-validate things all the time
- need to establish invariants

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Main lemma = subject reduction (also called preservation, etc.)

$$\Gamma \vdash T \wedge T \rightsquigarrow^\star T' \Rightarrow \Gamma \vdash T \cong T'$$

Why is SR hard?

Generation: eventually, you hit a proper structural rule

Have
$$\dfrac{\dfrac{\Gamma, x{:}\,A' \vdash t : B' \qquad \Gamma \vdash \Pi\, x{:}\,A'.B' \cong \Pi\, x{:}\,A.B}{\Gamma \vdash \lambda\, x.t : \Pi\, x{:}\,A.B} \qquad \Gamma \vdash u : A}{\Gamma \vdash (\lambda\, x.t)\, u : B[u]}$$

Want
$$\dfrac{\Gamma, x{:}\,A \vdash t : B \qquad \Gamma \vdash u : A}{\Gamma \vdash (\lambda\, x.t)\, u \cong t[u] : B[u]}$$

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$$\text{Have}\quad \frac{\dfrac{\Gamma, x{:}\,A' \vdash t : B' \qquad \Gamma \vdash \Pi\, x{:}\,A'.B' \cong \Pi\, x{:}\,A.B}{\Gamma \vdash \lambda\, x.t : \Pi\, x{:}\,A.B} \qquad \Gamma \vdash u : A}{\Gamma \vdash (\lambda\, x.t)\, u : B[u]}$$

$$\text{Want}\quad \frac{\Gamma, x{:}\,A \vdash t : B \qquad \Gamma \vdash u : A}{\Gamma \vdash (\lambda\, x.t)\, u \cong t[u] : B[u]}$$

Note: only uses injectivity, no-confusion is used for progress: every term is a whnf or can reduce (ie there are no weird stuck things like $\pi_1\ (\lambda\, x.t)$ or $0\ u$)

Invariant for neutral inference

$$\Gamma \vdash n \approx n' \triangleright A$$

What are the invariants?

Precondition Γ is well-formed; n, n' are well-typed (but **not necessarily the same type**)

Postcondition $\Gamma \vdash n \cong n' : A$ **and** A **is the most general/unique type of** n

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Also needs injectivity...

Positive soundness is already tricky

Negative soundness

Show every algorithmic step is **invertible**: the set of premises is necessary for the conclusion
If we reach a contradiction, the original problem also had no solution

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Show every algorithmic step is **invertible**: the set of premises is necessary for the conclusion
If we reach a contradiction, the original problem also had no solution

Won't show it again, but invariants all over the place

But now also injectivity for terms: all the congruences we apply are invertible!

Neutral injectivity

Injectivity of neutral eliminators?

If $\Gamma \vdash n \cong n' : T$ and n and n' are neutrals, then

- $n = x = n'$
- or $n = m\ u$, $n' = m'\ u'$ with $\Gamma \vdash m \cong m' : \Pi\, x{:}A.B$ and $\Gamma \vdash u \cong u' : A$
- or...

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- or...

Does not always hold!

$$x, y{:}(\mathbb{N} \to \top) \times \top \vdash x \cong y : (\mathbb{N} \to \top) \times \top$$

Injectivity of neutral eliminators?

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$$x, y{:}(\mathbb{N} \to \top) \times \top \vdash x \cong y : (\mathbb{N} \to \top) \times \top$$

Neutral comparison is invertible only at certain types

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$$x, y{:}(\mathbb{N} \to \top) \times \top \vdash x \cong y : (\mathbb{N} \to \top) \times \top$$

Neutral comparison is invertible only at certain types

Agda	Lean	Rocq
Type-directed	Term-directed	Term-directed
Short path for neutral functions	Detect unit-like types	Forbid unit-like types

	Positive soundness	Negative soundness (typed conversion)	Negative soundness (untyped conversion)	Termination
Injectivity of type constructors	×	×	×	×
Term-level injectivities		×*	×*	
Normalisation				×

* Subtlety for neutrals

	Positive soundness	Negative soundness (typed conversion)	Negative soundness (untyped conversion)	Termination
Injectivity of type constructors	×	×	×	×
Term-level injectivities		×*	×*	
Normalisation				×

Claim/conjecture: this analysis scales to realistic proof assistant kernels

How to prove the properties?

	Logical relation [AÖV17; PMT26]	**Rewriting/Confluence** [Tak95]/MetaRocq	**Gluing/Nf Model** [Ste21; BKS23]	**Domain model** [CH18]
Syntax	Raw	Raw	Intrinsic	Raw (Intrinsic?)
Weak ambiant th.	×	✓	×	✓
Normalisat°	✓	×	✓	×
Scaling	×	✓	?	?
η laws	✓	×	✓	✓
	Most explored	Insane scaling	Formalisation currently difficult	Very unexplored

Thank you!

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